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ABSTRACT

This guide includes a chapter on concepts related to measurement, a chapter describing the various measuring instruments that are used in the field, and a chapter indicating realistic class projects using field instruments. Twenty-three plates contained in the publication are designed as masters to make transparencies for classroom instruction. A bibliography of 28 source books is included. (Author/DT)

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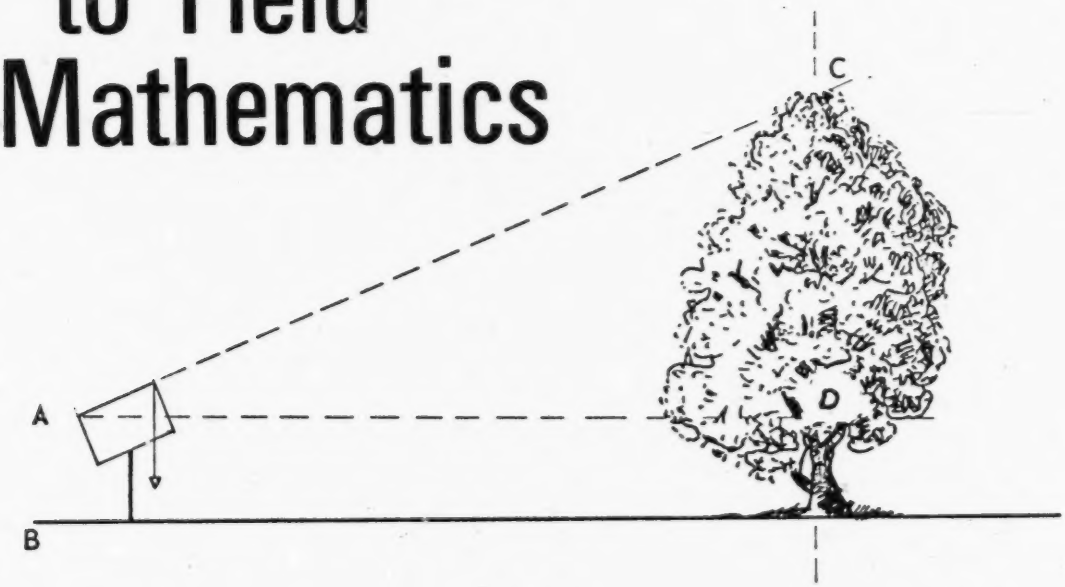
A Guide to Field Mathematics



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A Guide to Field Mathematics

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Bureau of General and Academic Education
Pennsylvania Department of Education
1972

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FOREWORD

This guide is designed to introduce mathematics teachers to field instruments and their use as instructional aids. It includes a chapter on concepts related to measurement, a chapter describing the various measuring instruments that are used in the field and a chapter indicating realistic class projects using field instruments. The 23 plates contained in the publication are designed as masters to make transparencies for classroom instruction.

Field instruments are an exceptional vehicle for mathematics teachers to make their subject meaningful and enjoyable and at the same time to provide useful mathematics. They permit data collection, decision making and problem solving as well as a multitude of mathematical concepts needed in the world of work and play. Furthermore, they help to eliminate separation "between things academic and things vocation," a goal of career education as described by the U.S. Commissioner of Education.

Selected field instruments can be integrated into the upper elementary school and middle school mathematics curriculum while these and other instruments are appropriate for inclusion into general mathematics, geometry and trigonometry courses. An alternative to this approach would be a senior high school level minicourse in field mathematics for which this guide could serve as a basis.

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INTRODUCTION

Why field work?

Mathematics is a rigorous, abstract, postulational system that could be studied for its beauty, but should it? For a small portion of the population this may be reason enough; but for the great majority of the population mathematics is only a tool to be used as necessary in conducting the business of living. For these people, using a measuring device is much more useful than proving theorems in topology and calculating gas mileage is more helpful than the development of a 9 point geometry. While it is true that an appreciation of mathematics is a desirable attribute that should be nurtured by the high school curriculum, pure mathematics is not necessarily the way to accomplish that goal.

A great bulk of mathematics has developed from observations and applications, ie. first the general notion and then the formal proof. Learning theorists indicate that this seems to be the same way that learning occurs. The learner should first explore and manipulate specifically designed devices to discover concepts and then proceed from these concrete experiences to more abstract models. Field mathematics can provide these experiences. Roger Bacon, in the thirteenth century, indicated a need for such instruments, observation and experiments in mathematics while 700 years later Dr. George Anderson, mathematics professor at Millersville State College, noted that field and laboratory work is essential to effective teaching of secondary mathematics. Anderson further observed that concepts

associated with instruments or other special mathematical devices are not easily forgotten.

Professor Lauren Woodby of Michigan State noted in the Cambridge Conference on School Mathematics that he was more successful with culturally deprived urban children when actually working with shadows and sticks for measuring heights than he was with above-average suburban children in a chalkboard discussion of shadows as a method of height measurement.

This guide is designed to introduce the classroom teacher to a number of field instruments that have been successfully used for many years in high school mathematics classes. A brief description of each instrument is given along with instructions regarding its use. There is a chapter on concepts involved in measurement such as accuracy, precision and significant digits. Students not familiar with these concepts would benefit from some instruction relevant to them. Possibly several measurement experiences should preceed such concepts as significant digits and rules for rounding, permitting the students to discover these concepts when performing related calculations.

Suggested projects for the student are included in the guide. The number of possible projects are limited only by the location of the school and the imagination of the teacher. Further suggestions may be found from books and articles listed in the bibliography.

A number of the plates in this guide are printed so that the page may be torn out and used in a copy machine to produce a transparency suitable for use on the overhead projector.

There is little doubt that the instruments discussed in this guide can be valuable teaching aids. The decision is left to the classroom teacher and the curriculum specialist to determine those devices that are most appropriate at each grade level. The method of presentation is also a variable. For some classes and some concepts a guided discovery method may be superior while in other classes and with other concepts, expository procedures may be more efficient. This decision is also left to the teacher.

MEASUREMENT

Webster defines 'measure' as the act or process of ascertaining the extent, dimensions or quantity of something by comparison with a standard. This can be contrasted to another method of determining a quantity called counting in which a 1-1 correspondence is established between the items under observation and the set of positive integers. If a quantity consists of a set of discrete elements that can be counted we say the set is countable or denumerable.

Example: Committee A contains three members.

Joe - 1

Sue - 2

Janet - 3

If the quantity to be discussed is a set of elements that is infinite but countable, ie. if we can establish a 1-1 correspondence between the positive integers and this set of elements, then we can say the set is denumerably infinite and we assign to it the symbol $\aleph_0$

read aleph-null. The set of rational numbers is such a denumerably infinite set.

It is possible to have a quantity consisting of an infinite set of elements that cannot be placed into a 1-1 correspondence with the positive integers. Such a set is called an uncountably infinite set. An example of this type of set would be the real numbers between 0 and 1. The sets described by 1, 2, and 4 below are countable while 3 is not countable.

1. grains of sand on Coney Island
2. the even integers
3. the irrationals between 10 and 11
4. the cells in a human body

Many quantities of interest do not consist of a set of elements necessitating an alternative method, called measuring, of expressing its quantity. A standard unit must first be agreed upon. This unit can then be used as a standard with which to compare or measure quantities of interest. Early attempts at defining a standard included the reigning king's foot or arm, grains of wheat, the pace of a local leader and many other rather unsatisfactory units. The evolution of these units has now reached the stage where the standard for time is in terms of radioactive decay and the standard for length is in terms of the wave-length of the orange-red line of the spectrum of Krypton 86. This is quite an improvement, for now a well equipped laboratory can duplicate its own standard. There is a very interesting chart on measurement standards called Modernized Metric System, available from the Superintendent of Documents, Government Printing Office, Washington, D.C. 20402.

A measurement system can be developed in the classroom if proper guidance is provided. Frequently this is done by the science teachers beginning with a meter for the standard unit of length and proceeding through units for area and volume. Prefixes are explained and linear measurements are made using meter sticks, micrometers and vernier calipers. Standard volume units are defined in terms of the linear units (1 liter = 1000 cubic centimeters, 1 ml = 1 cc) and students practice making these volume measurements using a graduated cylinder. Weight is defined in terms of volume (1 kilogram is the weight of 1 liter of pure water at $4^{o}C$ making 1 gram the weight of 1 cc of water), and again the students practice making measurements, this time using a balance scale. Other topics such as levers (balance scale), density (why water at $4^{o}C$ for standard weight?)*, verniers, mass, force and various measuring instruments can be integrated into measurement experiments.

This same organization and method of presentation can be effectively used by mathematics teachers at all levels in the secondary school. The depth of coverage will depend upon the grade level and ability of the class. A high school physics book is a good reference for this material.

An alternative to beginning with the meter as a standard unit of length would be to have the class decide upon a unit. Possible choices might be a pen, a pencil, a shoe or a stick. The students can be very creative when suggesting units and also when naming them.

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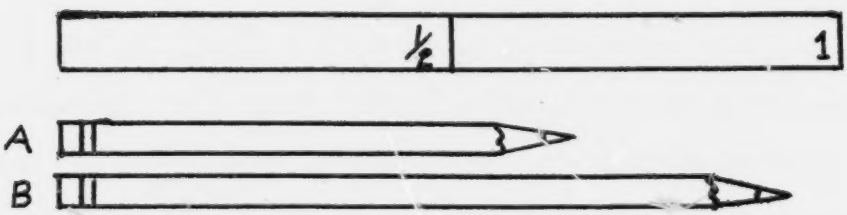
*The density of a substance is defined the ratio of its weight to its volume. It varies with the temperature of the substance and in the case of water the density is at a maximum at $4^{o}C$.

With the standard decided upon, various articles can be measured, such as a book, desk top, student height, room dimensions or distances between buildings.

The class will soon realize that finer measurements are not possible without dividing their standard unit into sub units. To demonstrate sub units a strip of transparency paper, to be used on an overhead projector, can be cut to represent the standard unit. A mark at its midpoint will make the finest unit on the measurement device one-half unit.

Here the notion of greatest possible error can be introduced, pointing out that if the item to be measured is less than half way between the 1/2 unit mark and the 1 unit mark we will call it 1/2 unit and our greatest error will be 1/4 unit. (See pencil A).



However if the item to be measured is more than half way between 1/2 unit and 1 unit then we will call it one unit (See pencil B). Smaller divisions can be made on the standard unit and by recording these values in a table it will be apparent that the greatest possible error is half of the smallest division on the measuring instrument. It should be noted that in every case, no matter how small the divisions on the

instrument, the measurement is approximate, thus there is always an error. The precision of the measure is simply the smallest division used on our instrument. In the case just discussed the precision is 1/2 unit. If the unit were divided into tenths then the precision would be 1/10 unit and measures made with the latter instrument would be more precise. The advantages and disadvantages of using divisions of tenths and sixteenths could be explored during this development.

An interesting case arises when measures are taken using instruments not having the same precision, for example one measure with an instrument marked in meters, the other measure with an instrument marked in centimeters. Suppose the distance from California to Pennsylvania is measured using a device marked in meters and is found to be 5,161,425 meters, while the width of a stamp is found to 3 cm using the other instrument.

The second measure would be more precise but certainly to measure the distance from California to Pennsylvania to the nearest meter is a considerable undertaking and has a quality that is superior to measuring the width of a stamp to the closest centimeter. This quality is called accuracy or relative error. It is simply the ratio of the maximum possible error to the measurement itself.

For the example just given, the accuracy of the California to Pennsylvania measurement is $\frac{0.5}{5,161,425}$ = .00000[illegible]9 or .00000969%, while the accuracy of the stamp measurement is $\frac{.5}{3}$ = .1666 or 17%. The relative error (accuracy) of .00000969% is much smaller than 17%, and therefore it identifies the more accurate measure. (Note

that the smaller of these two ratios is the lesser relative error, thus it represents the greater accuracy).

The notion of significant digits and rounding off approximate numbers should be explored while discussing measurements. With respect to rounding off it should be emphasized that the process does not consist of rounding off digit by digit starting at the right, but simply begins with the first digit to be dropped.

Example: Round to the nearest inch 7.468". This number is less than half way from 7" to 8", thus we call it 7". If we erroneously started at the right we would have successively 7.47", 7.5", and finally 8".

A further concern arises if the number to be dropped is 5. The convention:

> If the digit following the final digit to be retained is less than five, the final digit will not be altered but if it is greater than five, the final digit to be retained is increased by one.

does not consider the case where a five is to be dropped. Ideally, if it is less than five, it is dropped; if it is greater than five, it is dropped and one is added to the preceding digit; and if it is exactly five, e.g. round 9.5 to the nearest inch, add one to the preceding digit 50% of the time. This way, in the long run, the round off errors can be minimized, for if a set of these numbers were to be summed after they were rounded, and in every case where a five was to be dropped, a one was added

to the preceding number, the sum would be too great, and if we never added a one when we dropped a five, the sum would be too small.

Example:	(a) 6" rounded 6"	(b) 6" rounded 6"	(c) 6" rounded 6"
	4.5" rounded 5"	4.5" rounded 4"	4.5" rounded 4"
	5.5" rounded 6"	5.5" rounded 5"	5.5" rounded 6"
Sum:	17"	15"	16"

In example (c) we have minimized the round off errors by adding one, 50% of the time.

There are various strategies that can be used to add one about 50% of the time when dropping a 5.

1. On Tuesdays, Thursdays and Saturdays add one--Mondays, Wednesdays and Fridays do not add one--and Sundays do no mathematics.

2. Flip a coin, heads add one; tails do not add one.

3. Even numbered dates add one, odd numbered dates do not add one.

4. If the first digit on the right to be kept is odd, add one (making it even); if it is even do not add one (keeping it even). Example: 7.5" rounds to 8"
   6.5" rounds to 6"

At this point significant digits and scientific notation can be profitably introduced. As it was previously noted, all numbers that come from measurements are approximate. Each time a measuring device is used, the resulting measure has an error, the greatest error being one half of the smallest unit used. Approximate numbers can also come from rounding off exact numbers. For instance if the

student body of a university numbers 13,328, it may be rounded to 13,000 for some purposes. Thus 13,000 is an approximate number. Entries in tables of logarithms and trigonometric functions are other examples of numbers that are approximate but not obtained from measurements.

If the number comes from a carefully made measurement, then all of the digits are significant. Every digit is exact except the last and it is in error at most one half of the unit of measurement.

TABLE I

Example:	Unit Used	Number of Units in Sample Measure	Usually Written	Significant Digits	Scientific Notation
a.	1 inch	23	23 inches	2 and 3	2.3×10^{1}
b.	1 inch	20	20 inches	2 and 0	2.0×10^{1}
c.	1/10 inch	13	1.3 inches	1 and 3	1.3×10^{0}
d.	1/100 inch	245	2.45 inches	2, 4 and 5	2.45×10^{0}
e.	1/10 inch	4	.4 inches	4	4×10^{-1}
f.	1/100 inch	4	.04 inches	4	4×10^{-2}
g.	1/100 inch	204	2.04 inches	2, 0 and 4	2.04×10^{0}
h.	1/100 inch	40	.40 inches	4 and 0	4.0×10^{-1}

In example (b) and (h) in Table I the zeros are significant while in example (f) the zero is not. In (b) and (h) there are 20 and 40 units respectively but in (f) we have only 4 units. It is obvious the zero in (b) cannot be dropped for this would leave the numeral 2. It is less obvious but almost as disastrous to drop the zero in (h) and write .4 inches for this indicates a precision of 1/10 inch while the true precision is 1/100 inch.

Some authors recommend putting the number in scientific notation, ($a \times 10^k$, where $1 < a < 10$, and k is an integer), then all of the digits contained in "a" are significant. (See table 1).

It is important that students be given the opportunity to make measurements and do calculations using their measures. Calculations using real data are much more interesting and meaningful than text-book data. Generally instruments can be borrowed from the science, industrial arts and home economics departments for this purpose. The following list suggests a few of these.

1. Micrometer
2. Therometer (F and C)
3. Balance scale
4. Graduated cylinder
5. Calorimeter
6. Teaspoon, tablespoon, cup, quart
7. Vernier caliper
8. Linear expansion apparatus
9. Meter stick
10. Plasti-gage
11. Stopwatch and clock
12. Protractor
13. Scale (architects or engineers)
14. Calendar
15. Tire pressure gauge
16. Tire tread depth gauge
17. Angle mirror
18. Hypsometer
19. Clinometer
20. Transit
21. Sextant

Consideration of the accuracy and precision of the measures is necessary when subjecting them to arithmetic operations and the following rules should be presented and discussed.

Rule I. When adding or subtracting approximate numbers the sum or difference should have the same precision as the least precise of the numbers involved.

Example: (a) Add

$$\begin{array}{r} 45.4'' \\ 1.324'' \\ \hline 46.724'' \end{array} \approx 46.7$$

(b) Add

$$\begin{array}{r} 2.16'' \\ .0041'' \\ \hline 2.1641'' \end{array} \approx 2.16$$

(c) Subtract

$$\begin{array}{r} 54.157 \\ 2.31 \\ \hline 51.847 \end{array} \approx 51.85$$

This procedure is necessary since the digits eliminated are not significant. In example (a) for instance, the number 45.4 could be as large as 45.44 or as small as 45.36 and it would still be called 45.4 if measured with an instrument marked in tenths of an inch. The sum could then be as small as 45.36 + 1.324 = 46.684 or as large as 45.44 + 1.324 = 46.764, thus to say 45.4 + 1.234 = 46.724 would be signifying a precision of .001 yet the sum is actually in the interval 46.684 to 46.764.

Rule II. When multiplying or dividing approximate numbers the product or quotient should have no more significant digits than the least number of significant digits

contained by the numbers involved.

Example: (a) $3.41''$
$23\quad''$
1021
682
$78.21 \approx 78$ sq. in.

(b) $\frac{58.42}{.2} = 292.1 \approx 300$

(c) 143.2
$.001$
1432
0000
0000
$001.432 \approx 1$

(d) $20{,}000$
1.34
80000
60000
20000
$26800.00 \approx 30{,}000$

Example (d) may not or may be rounded correctly. If both numbers are approximate, the precision of 20,000 is not clear and it was assumed to be 10,000. If, in fact, it was 1,000 or 100, then the product should have been rounded to 27,000 or 26,800 respectively.

Notice that .001 in example (c) contains only one significant digit therefore the product has only one significant digit.

An appreciation of the need for this rule can be noted in (b). The divisor .2 with a precision of one tenth could really be as small as .16 or as large as .24 and still be called .2. This would allow the quotient to range from $58.42 \div .16 = 365$ to $58.42 \div .24 = 247$, thus to call it 292.1 would be a misrepresentation.

It needs to be emphasized that if the numbers are exact, the above rules are unnecessary and all of the digits are significant.

Dimensional analysis, used extensively by science teachers and often ignored by mathematics teachers, is well suited for introduction at this point. It is a procedure of converting from one unit to another by multiplying by one. The number one has an infinity of

representations (2/2, 100%, 1 hr./60 min., 12 in./1 ft., 1 meter/ 100 cm, etc.) and the choice to be used will be such that the unit to be discarded will divide out (cancel) and the desired unit will remain.

Example: (a) change 4 ft. to inches

$$4 \text{ ft.} \times \frac{12 \text{ inches}}{1 \text{ ft.}} = 48 \text{ inches}$$

(b) change 60 $\frac{\text{miles}}{\text{hr.}}$ to feet per minute

$$\frac{60 \text{ miles}}{\text{hr.}} \times \frac{5280 \text{ ft.}}{1 \text{ mile}} = 60(5280) \text{ ft./hr.}$$

$$60(5280) \frac{\text{ft.}}{\text{hr.}} \times \frac{1 \text{ hr.}}{60 \text{ min.}} = \frac{60(5280)}{60} \text{ ft./min.}$$

$$\frac{60(5280)}{60} \frac{\text{ft.}}{\text{min.}} \times \frac{1 \text{ min.}}{60 \text{ sec.}} = \frac{60(5280)}{60(60)} \text{ ft./sec.} =$$

88 ft./sec.

(c) Find the weight in kg. of a cube of copper 10 cm. on a side. Volume = 10 cm x 10 cm x 10 cm = 1000 cm^3. Since density is defined by weight per unit volume and density is tabled, the volume times the density will equal the weight. (density of Cu. = 8.9 $\frac{gm_3}{cm}$)

$$1000 \text{ cm}^3 \times \frac{8.9 \text{ gm}}{\text{cm}^3} = 8900 \text{ gm}$$

$$8900 \text{ gm} \times \frac{1 \text{ kg}}{1000 \text{ gm}} = 8.9 \text{ km}$$

(d) What gravitational force in newtons acts on a 800 g. body? (9.8 m/sec^2 = Acceleration due to gravity)

Force = Mass x acceleration
$F = 800 \text{ g} \times 9.8 \text{ meter/sec}^2$.
$F = 7840 \frac{\text{gram meters}}{\text{sec.}^2}$

Note: A newton is a $\frac{\text{kilogram meter}}{\text{sec.}^2}$

$$F = 7840 \frac{\text{gm meters}}{\text{sec.}^2} \times \frac{1 \text{ kg}}{1000 \text{ gm}} = 7.840 \frac{\text{kg. M}}{\text{sec.}^2} = 7.840 \text{ newtons}$$

(e) Convert 13 oz. to grams
(1 oz. = 28.350 grams)

$$13 \text{ oz.} \times \frac{28.325 \text{ gm}}{1 \text{ oz.}} = 367.055 \text{ gm}$$

Dimensional analysis can be an invaluable aid in science courses. Frequently the student may not be certain of the procedure necessary to solve a problem but by manipulating the units first until the desired unit remains, the chances are that by substituting numbers for the units, the correct solution will be obtained.

Problems of conversion from the metric system to the English system and the English system to the metric system are well suited for dimensional analysis but they do not deserve too much attention since most laboratory work uses the metric system exclusively, making conversion unnecessary. A better tack when discussing the metric system might be to have students practice estimating the conversions using their height and weight, gasoline tank volume, etc.

1 lb$\approx$1/2 kg	1 inch$\approx$2 1/2 cm
1 kg $\approx$2 lbs	1 yd$\approx$1 meter
1 oz $\approx$30 gms	1 mile$\approx$1 1/2 kilometers
1 liter $\approx$1 qt	

INSTRUMENTS

Vernier

A most interesting and invaluable procedure for increasing the precision of linear measures was devised by Pierre Vernier. It consists of an attachment to the measuring instrument designed such that each sub unit on the instrument can be divided into finer sub units, the advantage being an increase in the number of subdivisions possible and therefore the precision. For instance between 2.1 and 2.2 on the scale below there would be room for at most one division

allowing a precision of one half of one twentieth or one half of five hundredths which is 5/200, but with the aid of a vernier, ten divisions between 2.1 and 2.2 are possible, increasing the precision to one half of one hundredth (1/2 x 1/100 = 1/200).

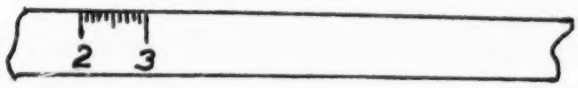


Plate 1 shows a unit being subdivided into fourths by a vernier. Fourths are seldom used but for the purpose of an introduction they seem superior to tenths.

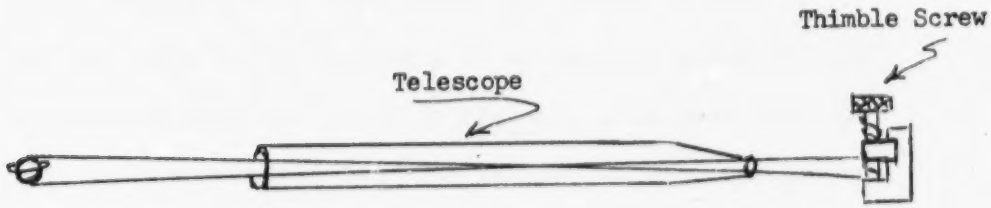
To read a vernier, the first digit in the measure is the numeral directly under the zero or if a numeral is not under the zero then the first numeral to the left of zero. The next digit in the measure is obtained from the vernier and read from the division on the vernier that is lined up with a division on the measuring instrument (if none are lined up use the one closest to being lined up).

From top to bottom in plate 1, the measures are 3, 3 1/4, 3 2/4 and 3 3/4. An explanation of why this works (see plate 2) is readily understood by high school students and can be easily presented using an overhead projector. Plate 2 may be used as a transparency master and by cutting out the vernier scale, various measures can be shown.

A development of a vernier dividing the units into tenths is outlined in plate 3 and into twelfths in plate 4. The vernier represented in plate 4 is used on the transit and will be needed when discussing field instruments.

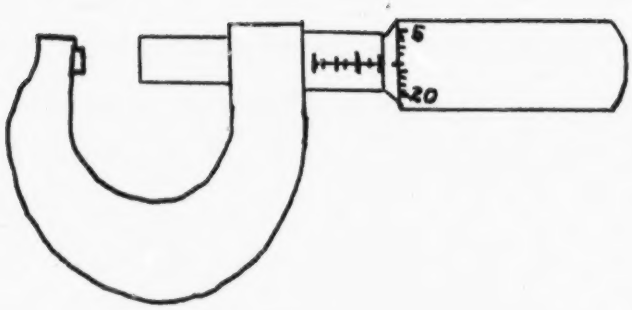
Micrometer

Another measuring instrument designed to increase precision was invented by a Frenchman by the name of Palmer using a discovery of the astronomer William Gascoigne. Gascoigne was interested in determining the diameter of planets by measuring the width of light from the planet, at the eyepiece of a telescope. His discovery consisted of a vise-like instrument with jaws operated by a thimble screw.



If there are 40 threads to the inch then one revolution of the thimble will move the screw, and therefore the jaws, 1/40 or .025 inch.

Palmer further developed this idea by engraving marks on the hub of the instrument resulting in what we now call a micrometer.



Four revolutions of the thimble represent 4 x .025 or .100. The thimble is marked with 25 equal divisions each representing 1/25 of one revolution or 1/25 of 25/1000 which is .001. The precision of the instrument is therefore .001 inch and the greatest possible error of a carefully made measurement is 1/2 of .001 or .0005 inch. To read a micrometer setting, first read the marks along the hub then add to this the reading of the thimble. See plate 5.

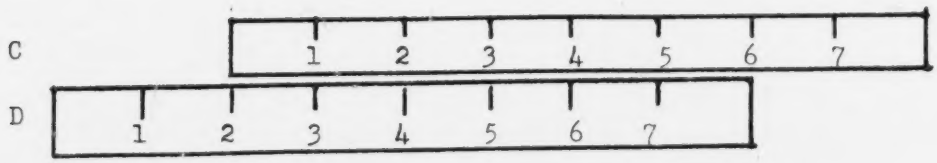
Practice using the micrometer (both metric and English) and vernier calipers as well as many other measuring instruments can be an excellent learning experience. There are measurement experiments that can be used at every educational level and are appropriate for discovery and self-learning techniques.

Slide Rule

The slide rule is nothing more than a measuring device. It can be introduced at many levels in the high school but it may be preferable to introduce it after a unit on logarithms. For those

students who never have had a mathematics course that included logarithms, the introduction of the slide rule should be preceded by some instruction on measurement, scales and logarithms.

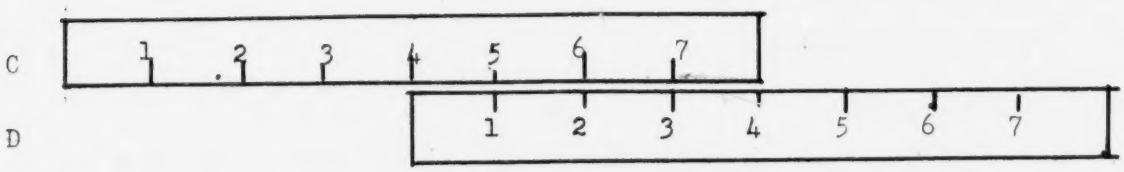
A simple adding machine can be constructed by using two rulers side by side, marked with the same equal size units.



In the figure above the left end of ruler C is over the 2 on ruler D and below 3 on ruler C a 5 may be read on ruler D. In effect we have added 2 to 3 by adding a length of two units on the D ruler or scale to a length of three units on the C scale and obtained a total length or sum of five.

If logarithmic scales were used in place of linear scales, the above procedure would add logarithms, a process that multiplies numbers.

The same method of development can be pursued for subtraction.



The problem of subtracting 3 from 7 is illustrated by putting the 7 on the C scale directly over the 3 on the D scale. The difference is 4, the overhang on the C scale. With logarithmic scales, the subtraction represents a division of the numbers.

The students can verbally assist in the development of the movable logarithmic scales via the overhead projector. Using a unit divided into ten sub-units, logarithms to the base 10 of the integers from one to ten can be located. (See plate 6.) It should be pointed out that the numbers on the finished scales really represent anti-logarithms since there is little need for the sum of the logs.

The non-linearity of the scales should be noted and for this reason the distance from 0 to 1 will have more subdivisions than say the distance from 8 to 9. Reading the scales is a stumbling block for many students and a visual approach may prevent mis-

understandings.

Following the above presentation the rules can be cut from the transparency and used with the overhead projector as an aid for instruction of slide rule operations. (Plate 6 can be used as a transparency master.)

Scale Drawing

A background in scale drawing will prove very helpful when attempting to diagram problems encountered in field mathematics.

Many examples of scale drawings may be found in the community. For instance some students may have access to the builders plans of their home or plans may be found in various magazines. Other examples might include maps and deeds.

Measurements using an architects' scale can be made on existing house plans or drawings may be made, given the dimensions of floor plans of homes or even of the school building. Deeds are another source of real data that can be used to make scale drawings.

A week or more could easily be spent on this topic and if professional scales are not available, plate 7 can be used with a copy machine to make a ditto master. Oak tag scales can be run off on a ditto machine and cut out for student use.

The Angle Mirror

The angle mirror is simply a device to determine and construct right angles. It is possible to have students manufacture their own angle mirror and from a pedagogical standpoint it may provide some student motivation, however the commercial instrument is quite

inexpensive and should not cause any hardship on the school budget.

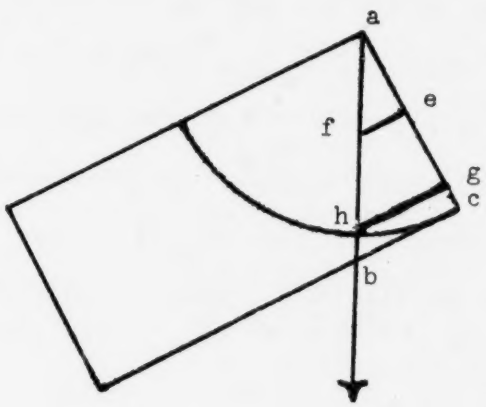
Plate 8 shows the approximate size of an angle mirror and also a schematic of the geometric principles involved in a 45^{o} instrument. Other variations use prisms and some are 30^{o} rather than 45^{o}. Plate 9 represents a 45^{o} angle mirror on a Jacobs staff. (A Jacobs staff is simply a 3 to 6 foot pole used to hold the angle mirror or the hypsometer.) The ranging pole K is sighted through the instrument from the point G. Ranging pole L is moved until its image is visible in the mirror at D and coincides with the view of K. When this situation is accomplished, the angle KEL is 90^{o}.

The Hypsometer

The hypsometer is another instrument that can be made by students. It is a piece of plywood or hardboard about 10" by 15" upon which a piece of 10 squares/inch graph paper is glued, (half inch graph paper may be more appropriate for lower grades). A screw is affixed at the upper right hand corner to which a plumb line is attached, (see plate 10), and a hole is drilled in the board for the bolt that will connect the hypsometer to the Jacobs staff.

To measure the height of an object the student sights the top of the object along the upper edge of the hypsometer, or through a sighting device if it is so equipped. The triangle formed by the plumb line, the right edge of the hypsometer and a portion of the bottom edge of the hypsometer, is similar to the triangle formed using as vertices the hypsometer and the top and bottom of the object to be measured, $\triangle ACB \sim \triangle abc$. Lengths $\overline{ac}$ and $\overline{bc}$ can be read directly and if AC is measured then BC can be found using a proportion. An alternative to using a proportion would be to scale the distance

$\overline{AC}$ along the line $\overline{ac}$. The triangle afe is similar to abc and to triangle ABC. Therefore $\overline{ef}$ will be the scaled distance BC. In either case the height of the hypsometer from the ground must be added to BC.



If $\overline{ac}$ is designed so that it contains 100 units then the sine of angle CAB, which is the same as angle cab, is the length $\overline{hg}$ while the tangent of this angle is the length $\overline{cb}$. (Plate 11)

The Clinometer

A clinometer is a device used to measure angles of elevation and depression. The hypsometer can be altered to serve as a clinometer by adding a zero to 90° arc as shown in plate 12.

The device is sighted in the same manner as the hypsometer and the angle is read where the plumb line crosses the arc.

The proof of the geometric foundations of this instrument is outlined at the bottom of plate 12, and a procedure to obtain trignometric functions from it are outlined on plate 11.

The Transit and Transit Level

The transit, the most sophisticated of the instruments discussed thus far, is designed to measure vertical and horizontal

angles. Mounted upon a single frame is a telescope, bubbles for leveling and two verniers to permit more precise angle measurement. (See plate 13).

When positioned upon a tripod the bubbles act as guides to level the transit by using four leveling screws. By turning one pair of opposite screws simultaneously (in opposing directions) the one bubble can be centered. The same procedure is used with the other pair of screws and the other bubble. If the instrument has only one bubble it must be leveled with the bubble in one position, then the bubble is rotated 90° and leveled again.

Once leveled, horizontal angles may be measured by locating one reference point in the cross hairs of the scope and setting the vernier on the graduated horizontal arc to zero. Then by sighting the other reference point in question the angle may be read on the horizontal vernier. Vertical angles are measured in a similar fashion.

Vertical and horizontal slow motion screws enable a fine adjustment to be made when sighting reference points.

The Plane Table

A simple and inexpensive method of mapping is called radiation and needs only a drawing board and an alidade, which is a straight edge with some sort of sighting arrangement. A sheet of drawing paper is attached to the plane table which is in turn mounted upon a tripod. The table is placed centrally in the area to be mapped (A in plate 14), leveled and a pin is placed in the center of the table. With the edge of the alidade against the pin, principle points are sighted in succession and lines are drawn radiating from

the pin at point A.

The actual distances from the plane table to each of the sighted points are measured and then scaled to allow each point to be determined on the map. The boundary of the area mapped is made by connecting the points so determined.

The Sextant

The sextant (plate 15) is a device to measure horizontal, vertical or inclined angles. It consists of a pair of mirrors, mounted on a frame, one of which is attached to an arm which in turn rides on a graduated arc. The principle is similar to the angle mirror but the angle between the mirrors may vary from 0 to 60° (thus the name sextant), measuring angles to 120°. If the arm is clamped at 90° (45° between the mirrors), the instrument can be used as an angle mirror.

To use the sextant one reference point is sighted through the sighting piece and by moving the arm until the image of the second reference point coincides with the first point, the angle measure can be read on the graduated arc.

SUGGESTED PROJECTS

Class Organization

Class organization and preparation prior to working in the field is critical to a successful learning experience. Projects must be well planned and described to the class with each student aware of the total task and of the part he is to play. Written instructions are often helpful and space may be provided on these instructions for

notes, drawings and measurements taken in the field.

Teams of three to six members each should be formed and a team leader identified. Each team should be assigned a specific project using certain equipment. If sufficient equipment is available each team can be using the same type of instruments simultaneously but the projects will have to vary to prevent the teams from interfering with each others measurements. If equipment is in short supply, each team will have to use a different type of instrument on a given day.

Several evaluative procedures are possible. For small exercises each member may produce a product while large projects such as surveying or map making a single team product could be acceptable with each team member receiving the same grade. Formal testing is optional.

Mathematics club members or better students might be allowed to set up some of the projects on their own such as the development of a deed, determining certain areas or finding inaccessible distances. These can then be assigned to other students as team projects.

Measurement of Heights

A. Shadow Method (plate 16)

Equipment: meter stick, 100 foot tape

Procedure: Hold the meter stick upright and measure the shadow $\overline{ab}$. Since $\triangle$ ABC and $\triangle$ abc are similar, measurement of $\overline{AB}$ will allow computation of the height of the tree $\overline{CB}$ using the proportion:

$$\frac{CB}{AB} = \frac{cb}{ab}$$

Scale drawing can be used in place of the proportion above.

B. Stadia (plate 16)

Equipment: meter stick, 100 foot tape

Procedure: By sighting along the meter stick the top and bottom of the tree, the points B and E can be located on the meter stick. Length BE can then be read and AE and AD can be measured. Since $\triangle ABE$ and $\triangle ACD$ are similar, CD can be calculated using the proportion:

$$\frac{AE}{BE} = \frac{AD}{CD}$$

C. Angle of Elevation (plate 12)

Equipment: 100 foot tape and clinometer or transit or sextant

Method: By measuring the angle of elevation of the top of the object and the distance of the base of the object from the instrument the height can be calculated using the tangent function:

$$\text{TAN} \angle CAD = \frac{CD}{AD}$$

$$\text{Height of tree} = CD + AB$$

An alternative to the tangent function would be to use scale drawing.

D. Angle Mirror (plate 17)

Equipment: angle mirror, 100 foot tape

Method: By positioning the angle mirror to allow the top and bottom of the building to coincide, the building will then form the hypotenuse of a right triangle.

Measuring the distance of the mirror from the building (CE) and the height of the mirror (CD), will permit calculation of the height of the building (AB) using the proportion:

$$\frac{AE}{CE} = \frac{CE}{BE}$$

since the altitude of a right triangle is the mean proportional between the intercepted segments.

Measurement of Inaccessible Distances

A. Angle Mirror

Equipment: angle mirror, 100 foot tape, ranging poles

Method: With the angle mirror at D (plate 18) have a student with a ranging pole locate a point B such that BDC is 90°. Then position the angle mirror at point B and locate a student at some point A on line AC. This makes angle CBA 90°. The distances BD and AD can be measured and using the proportion $\frac{CD}{BD} = \frac{BD}{AD}$ the distance CD can be found.

B. Transit - Sextant

Equipment: Transit or sextant, 100 foot tape, ranging pole

Method: Locate the transit or sextant at point A (plate 19). Locate some point C and then measure the angle CAB. Move the instrument to point C and measure angle ACB. Measure the length of AC and using the sine theorem $\frac{B}{\text{Sin } B} = \frac{C}{\text{Sin } C}$ solve for side C.

An alternative to the sine theorem would be to use a scale drawing.

Leveling

Equipment: A transit, hand level or clinometer and a leveling rod

Method: To find the difference in levels between two points, a transit, hand level or clinometer can be used. Plate 20 indicates such a process. The columns in the chart are 'Sta' for station, 'BS' for back sight, 'HI' for height of instrument, 'FS' for front sight and 'ELEV' for elevation. BM under "NOTES" indicates bench mark.

If the problem were to find the difference in elevation between point A and point B, the transit would be positioned between these two points and locked into a horizontal position. Assuming point A is known to be 60' above sea level (bench mark) and the back sight is 1 (i.e. the transit telescope is one foot higher than point A) and the front sight is 5, (i.e. the transit telescope is 5 feet above point B) then the difference in elevation between points A and B is 4 feet and the elevation of point B is 56 feet as noted in the second row of the chart under ELEV. If however, the distances between points or the elevation differences are too great, then a series of similar measurements in steps must be made. In this case the height of the instrument, HI, will be calculated at each step and in the final step the elevation of point F will be the elevation of the transit head at its last position (between E and F) less the front sight (FS) reading of point F to complete the chart. Each station has a row represented by a letter and the BS and FS represent the

back sight and front sight on that point or station. The transit height is the height of the transit when located such that it would back sight on that station. Eg: for station D, the transit is between D and E. The BS is the back sight upon station D and the FS is the front sight on station D when the transit was located between C and D. The height of the instrument, 61 feet, is the height of the instrument at its previous location, 58 feet, increased by BS-FS of the station in question which would be 6-3. Thus the transit height is 58 + (6-3) or 61 feet when located between point D and point E.

Deeds

One of the most frequent uses of the transit is to identify property boundary lines. This process can make a very interesting and educational project and can be structured in one of two ways.

1. Use an existing deed of the school property or of a nearby private plot.
2. Have a group of students, possibly a mathematics club, create a deed by locating reference points on the school property. Points such as home plate on the baseball field or the corner of a parking lot can be used as starting locations. From here the size and shape of the plot is limited only by the imagination of the students and the physical characteristics of the property. After the angles and dimensions are determined, the angles can be converted to north-south and east-west directions and a real-

istic deed can be written. Copies can then be given to class members for a surveying project.

To prevent the problem of having to determine true north, two reference points could be chosen that are easily recognizable. The final deed will then contain enough information to determine "true north" for the project.

Eg. From point A (first base on the baseball field) continue North $42^{\circ}15'$ East for a distance of 90' to a point B (home plate) thence South $74^{\circ}20'$ West, etc.

Another alternative for "true north" would be to indicate that "true north" is in the straight line which contains point A and the top of the flag pole or some other stationary landmark.

Maps and Areas

A. Plane table

Equipment: plane table, 100 foot tape, ranging pole, alidade

Method: Place the plane table centrally within the area to be mapped. Level the table, attach a sheet of drawing paper to the table and then place a pin near the center of the table (plate 14). By placing the alidade against the pin and sighting, in succession, each of the corners of the plot lines may be drawn radiating from the pin to each of the corners. The actual distances from the table to each of the corners can be measured and scaled down in order to locate the points on the drawing to complete the map.

The area of the map can be calculated using trianglations. An alternative would be to use graph paper rather than drawing paper to make the original map and then count the squares.

B. Angle mirror

Equipment: angle mirror, 100 ft. tape, ranging poles

Method: If the area to be determined is a plot of ground such as the upper figure on plate 21 the plan will be to divide the plot into trapezoids and triangles and then sum their areas. Lay out a working line WW' and divide it into equal segments. Then using the angle mirror, determine a right angle at A, then B . . . then H. The length of the perpendiculars to WW' at each of these points can be measured and are the bases of the trapezoids and triangles whose sum will approximate the area of the plot. If the area to be measured is inaccessible such as the lake on the bottom of plate 21 a similar procedure is used. First lay out a rectangle surrounding the lake WXYZ. Then using the angle mirror, determine perpendiculars to XY and WZ. Calculate the areas of the trapezoids formed within the rectangle but not including the water area. Sum these areas and subtract them from the area of rectangle WXYZ.

A FINAL WORD

This Guide has described a number of measuring instruments that can be used as instructional aids in teaching mathematics. Some of

instruments such as the angle mirror can be introduced to better students in the middle school, average students in the junior high school and all students in the senior high school. Concepts related to angle mirror projects range from the definition of a right angle through theorems from Euclidean geometry to the trapezoidal rule and Simpson's rule for finding areas. The other instruments have a similar range of related mathematical concepts.

Instruments designed for school use are quite inexpensive. The angle mirror, hypsometer-clinometer, vernier caliper, micrometer and hand level are all in the ten dollar range while the sextant and plane table can be purchased for under fifty dollars. A level transit suitable for instruction in field mathematics is available for about $150.

Students asking the question "why are we learning this mathematics?" will find an answer when using field instruments. Furthermore they will enjoy mathematics which will be a new experience for some of them.

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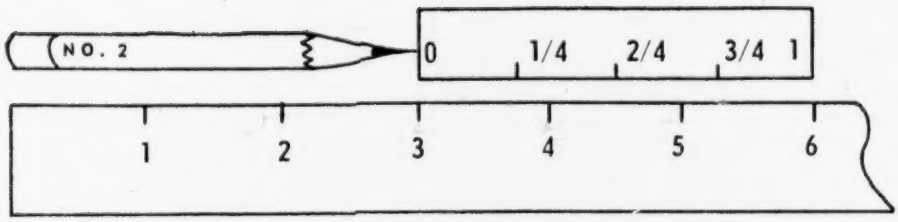
PIERRE VERNIER



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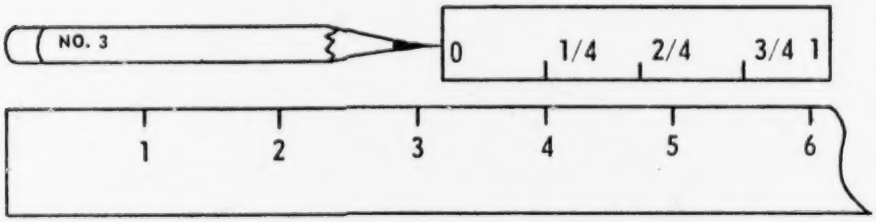
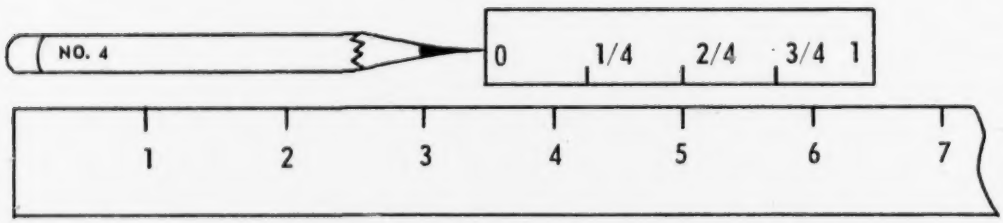
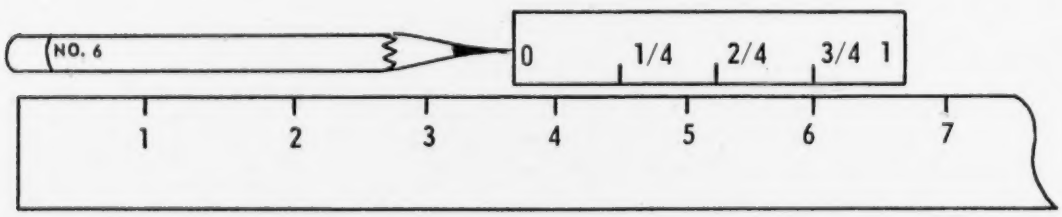


3 1/4

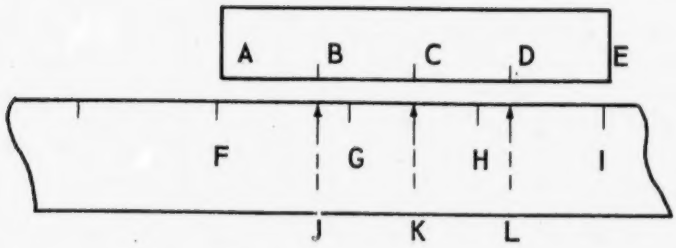


3 2/4



3 3/4

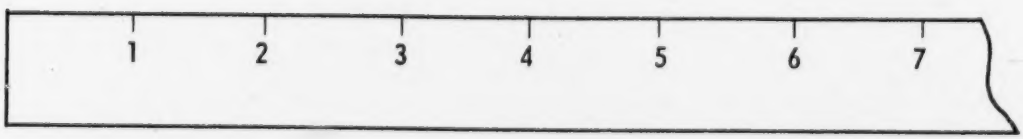
Plate 1

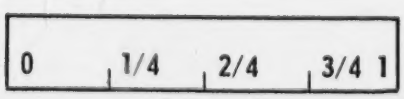


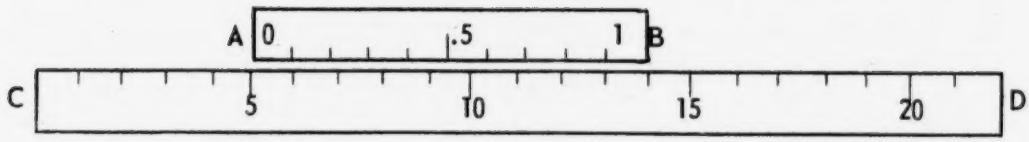
$AB = BC = CD = DE = x$
$FG = GH = HI = y$

$4x = 3y$
$x = 3/4$

$\therefore JG = 1/4\,y = 1/4\ FG$
$KH = 2/4\,y = 2/4\ GH$
$LI = 3/4\,y = 3/4\ HI$

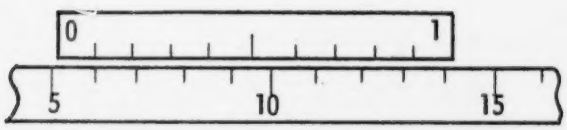




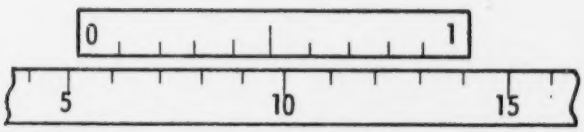


NOTE: LET EACH UNIT ON AB $= x$
AND EACH UNIT ON CD $= y$

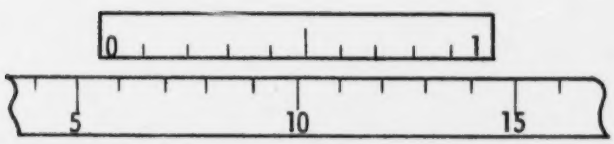
THEN $10x = 9y$
$x = 9/10y$



THUS IF VERNIER IS MOVED SO THAT .1 ON AB LINES UP WITH 6 ON CD, THEN IT MOVED 1/10 OF A CD UNIT.



IF MOVED SO THAT .2 ON AB LINES UP WITH 7 ON CD, THEN IT MOVED 2/10 OF A CD UNIT.



IF MOVED SO THAT .6 ON AB LINES UP WITH 11 ON CD, THEN IT MOVED 6/10 OF A CD UNIT.

ETC.

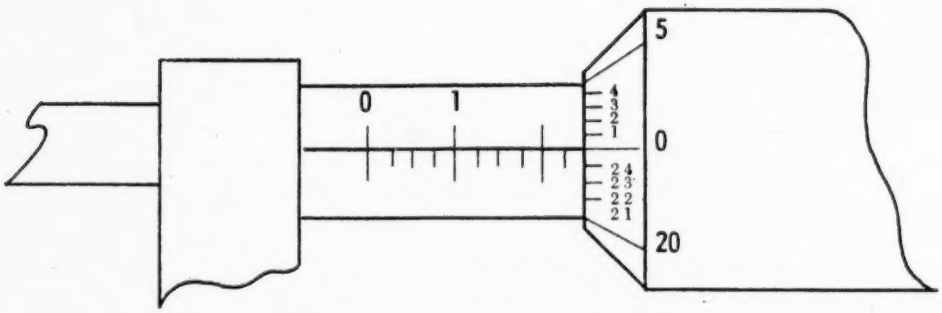
HERE WE WANT TO DIVIDE EACH UNIT ON THE DEGREE SCALE INTO 12 PARTS WHEREAS BEFORE THE GOAL WAS TO DIVIDE A UNIT INTO 4 PARTS OR 10 PARTS. THE PROCEDURE IS THE SAME. TAKE 11 UNITS ON THE SCALE AND REDIVIDE THEM INTO 12 PARTS.

$$12x = 11y$$
$$x = 11/12y$$

THUS EACH TIME A NEW DIVISION IS LINED UP, WE HAVE MOVED ANOTHER 1/12 OF OUR UNIT , AND 1/12 OF A DEGREE IS 5'.

NOTE: WHEN USING SCALE TO THE RIGHT OF ZERO, USE THE VERNIER TO THE RIGHT AND SCALE TO THE LEFT, VERNIER LEFT.

MICROMETER



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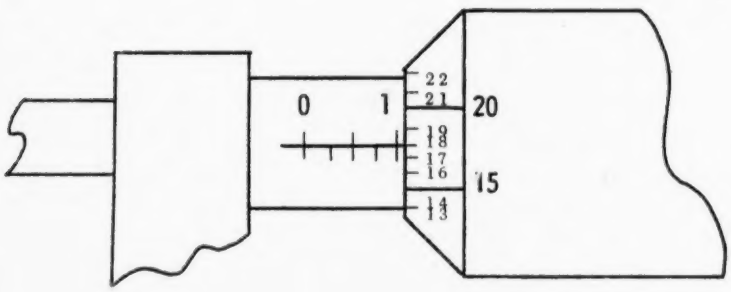

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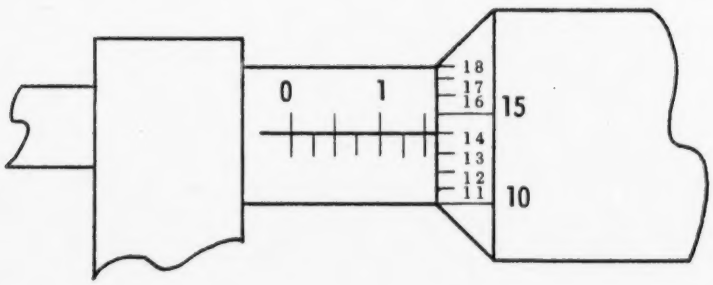
.018 INCH
.118 INCH

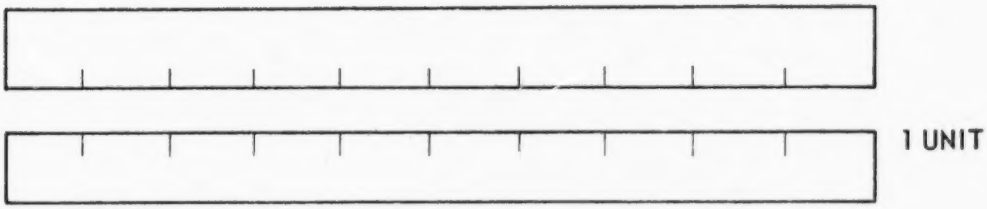


.150 INCH
.014 INCH
.164 INCH

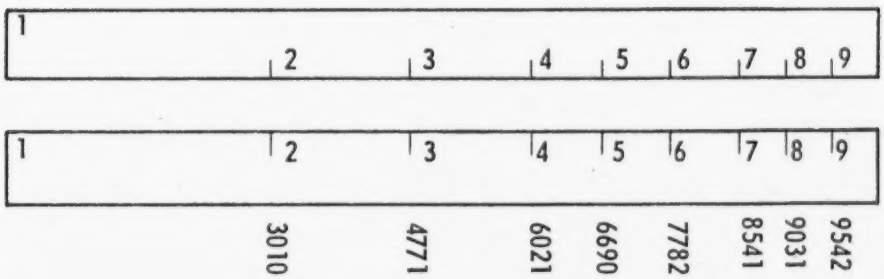
40 THREADS/INCH

1 REVOLUTION = 1/40 INCH = .025 INCH

SLIDE RULE



LOG 1 = .0000	LOG 6 = .7782
LOG 2 = .3010	LOG 7 = .8451
LOG 3 = .4771	LOG 8 = .9031
LOG 4 = .6021	LOG 9 = .9542
LOG 5 = .6990	LOG 10 = 1.0000



SCALE

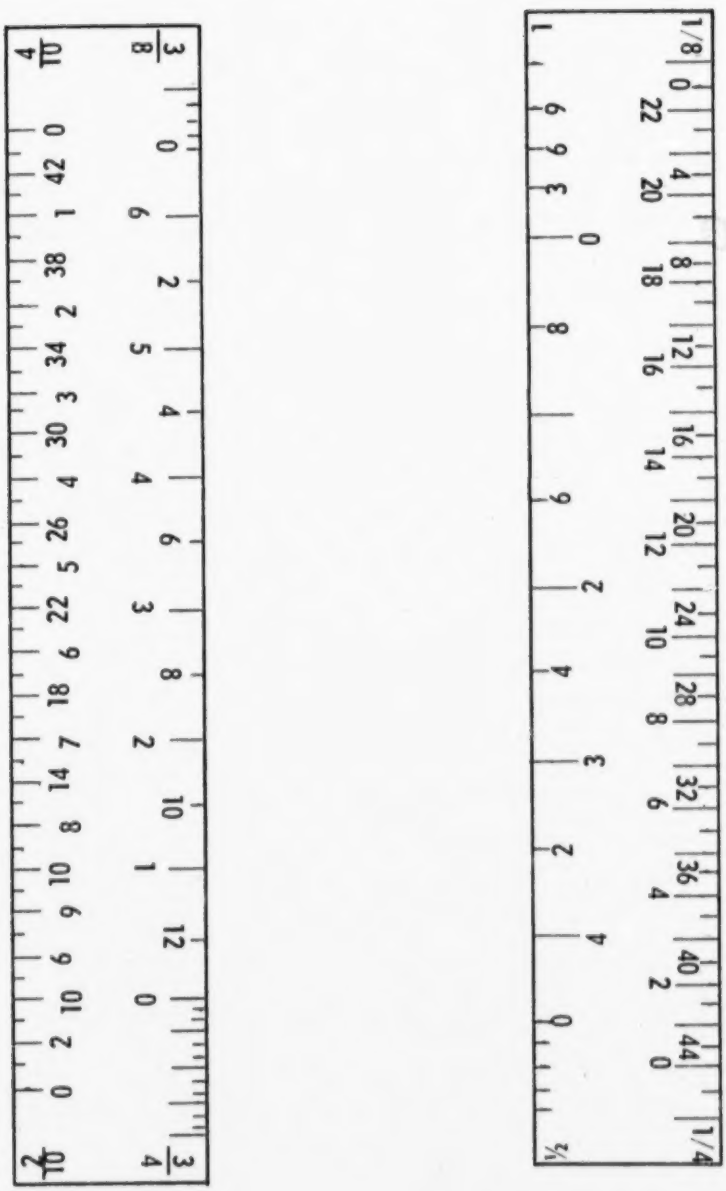
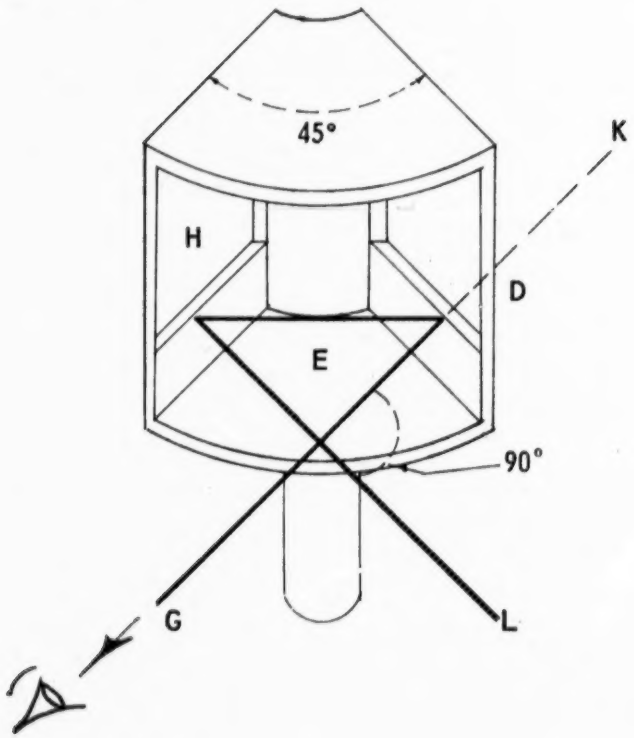
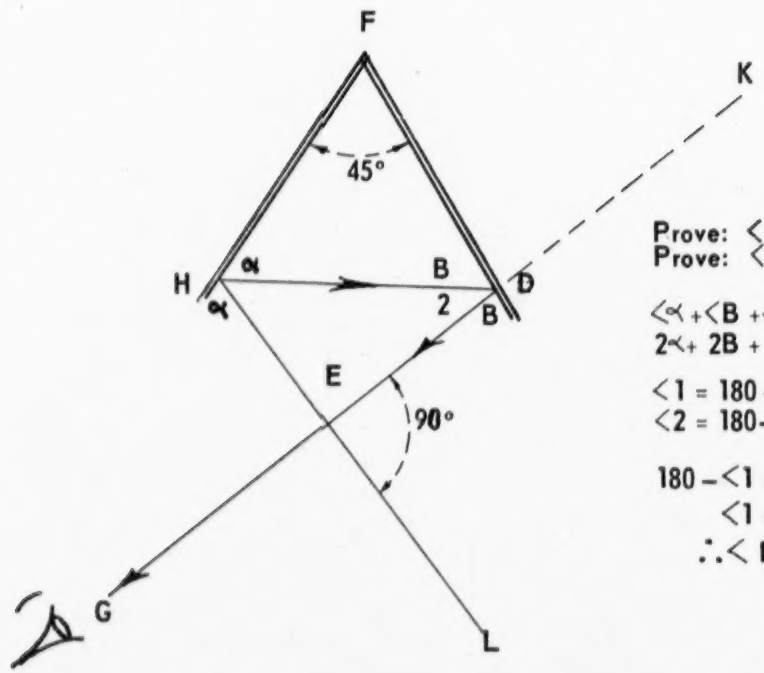


Plate 7

ANGLE MIRROR





Prove: $\angle KEL = 90°$
Prove: $\angle KEL = 2\angle F$

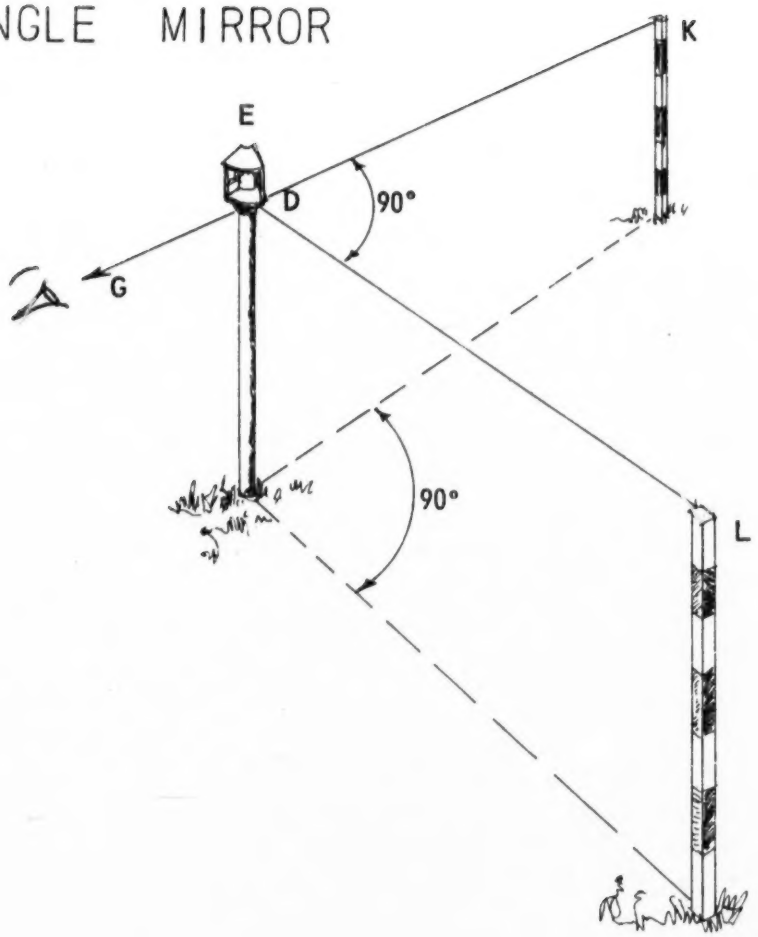
$\angle\alpha + \angle B + \angle F = 180°$
$2\angle\alpha + 2B + 2F = 360°$

$\angle 1 = 180 - 2\alpha$
$\angle 2 = 180 - 2B$

$180 - \angle 1 + 180 - \angle 2 + 2\angle F = 360$
$\angle 1 + \angle 2 = 2F$
$\therefore \angle DEL = 2F$

Plate 8

ANGLE MIRROR



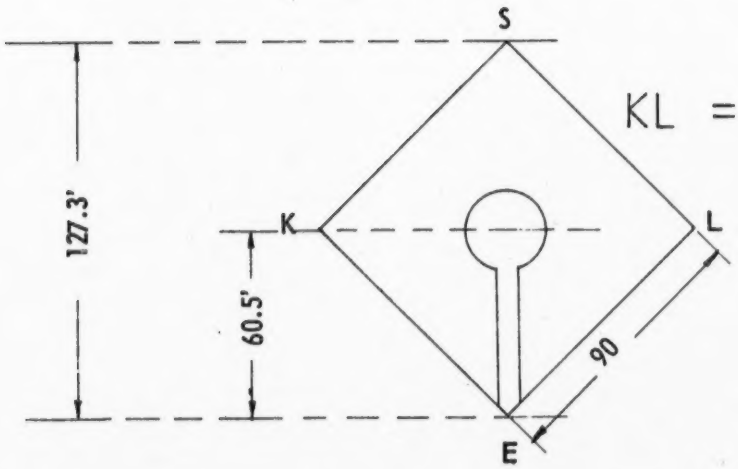
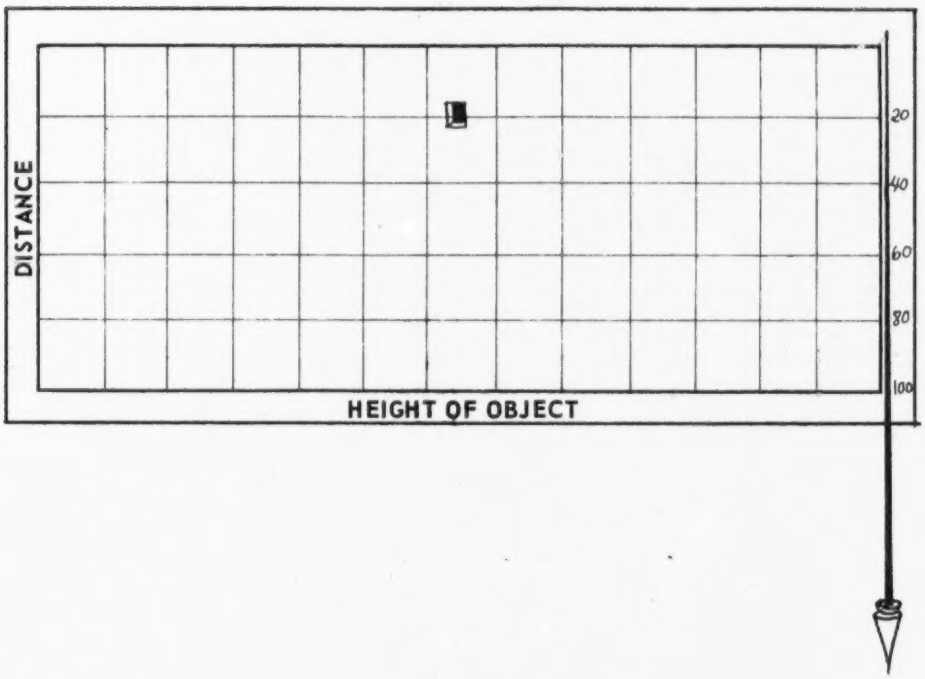


Plate 9

HYPSOMETER



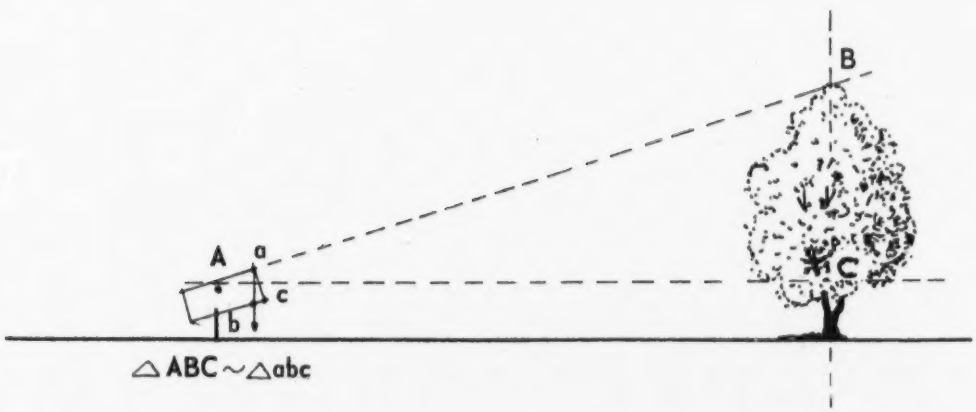
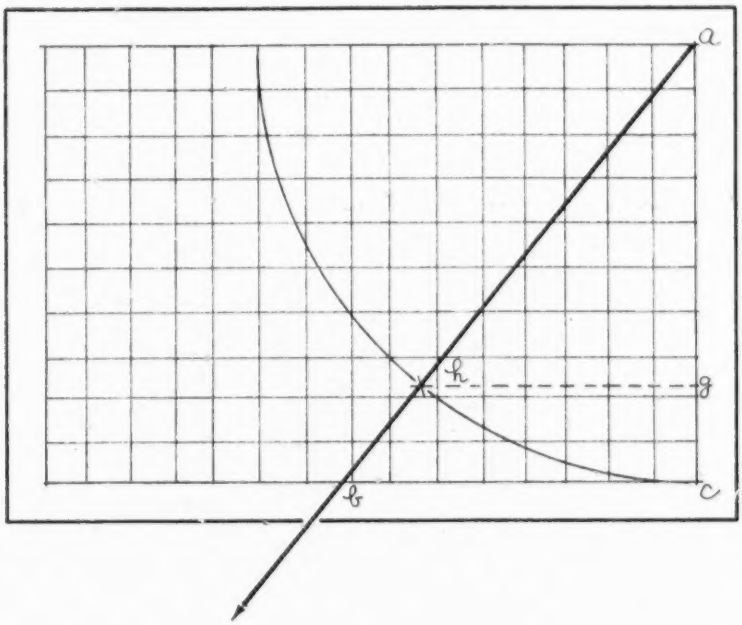


Plate 10

HYSOMETER AS A SINE, COSINE AND TANGENT TABLE

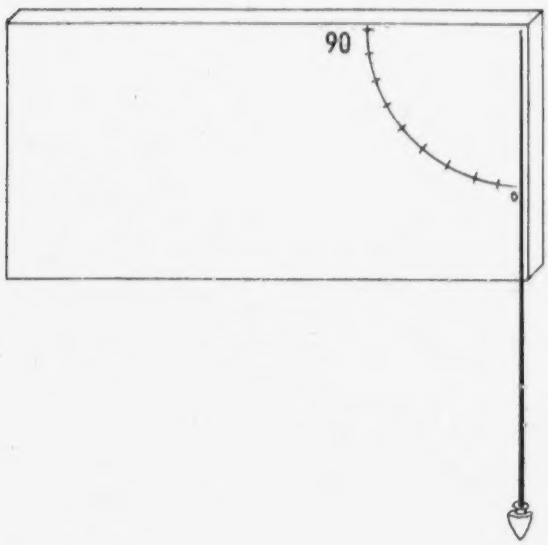


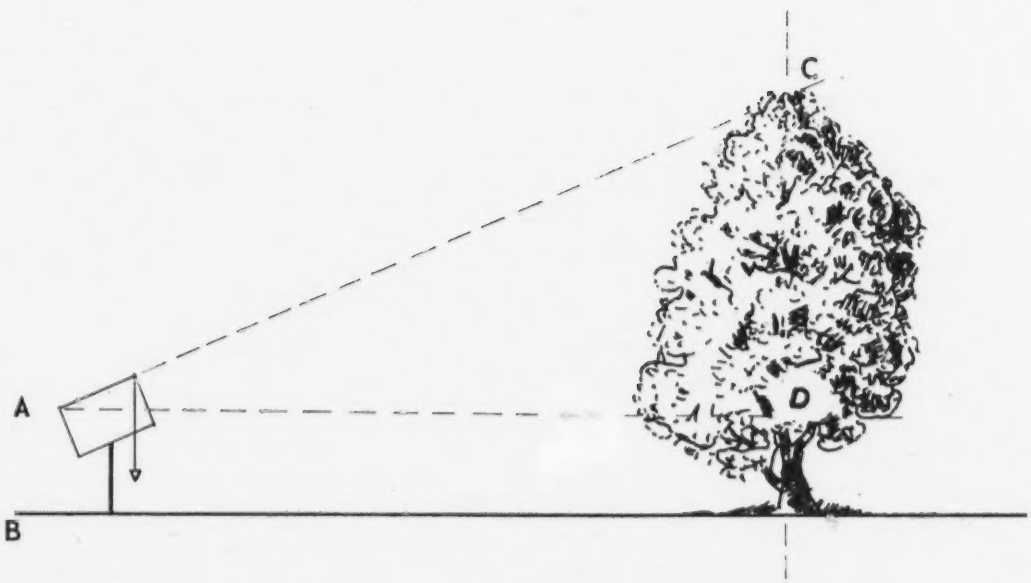
$$\text{SIN}\angle bac = \frac{hg}{ah} = \frac{hg}{1} = hg$$

$$\text{COS}\angle bac = \frac{ag}{ah} = \frac{ag}{1} = ag$$

$$\text{TAN}\angle bac = \frac{bc}{ac} = \frac{bc}{1} = bc$$

CLINOMETER





$\angle 1 + \angle 2 = 90$

$\angle 2 + \angle 3 = 90$

$\angle 1 = \angle 3$

AND $\angle CAD = \angle 3$

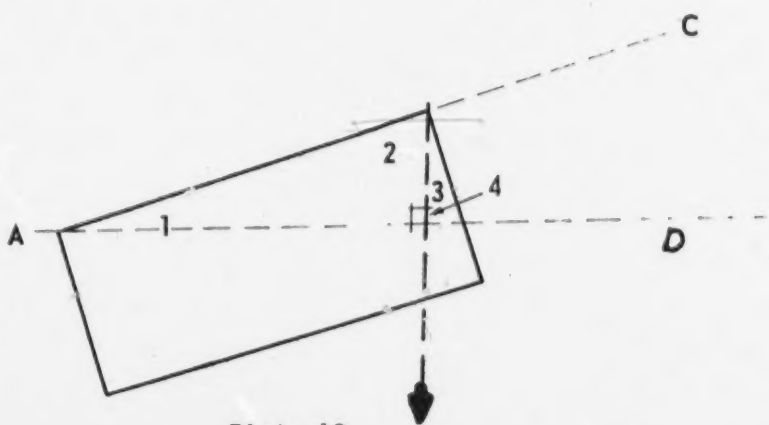


Plate 12

TRANSIT LEVEL

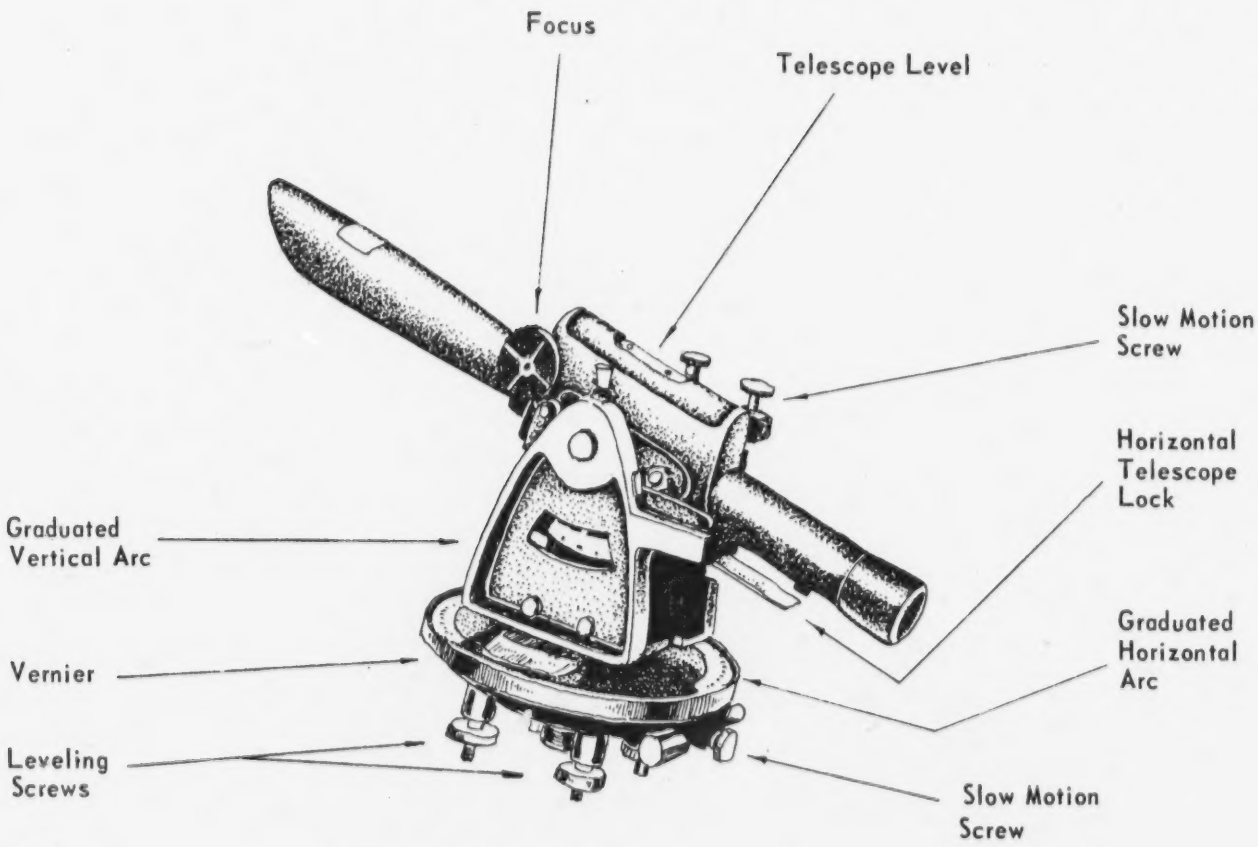
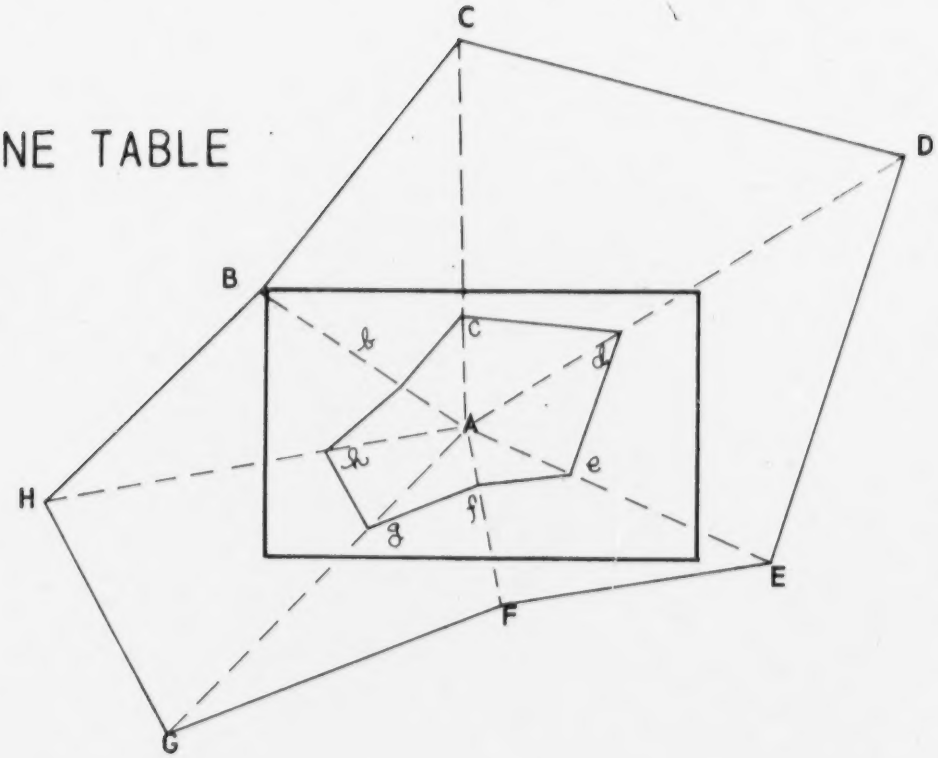
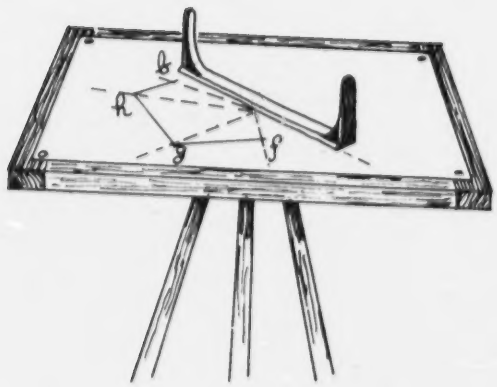


Plate 13

PLANE TABLE





SEXTANT

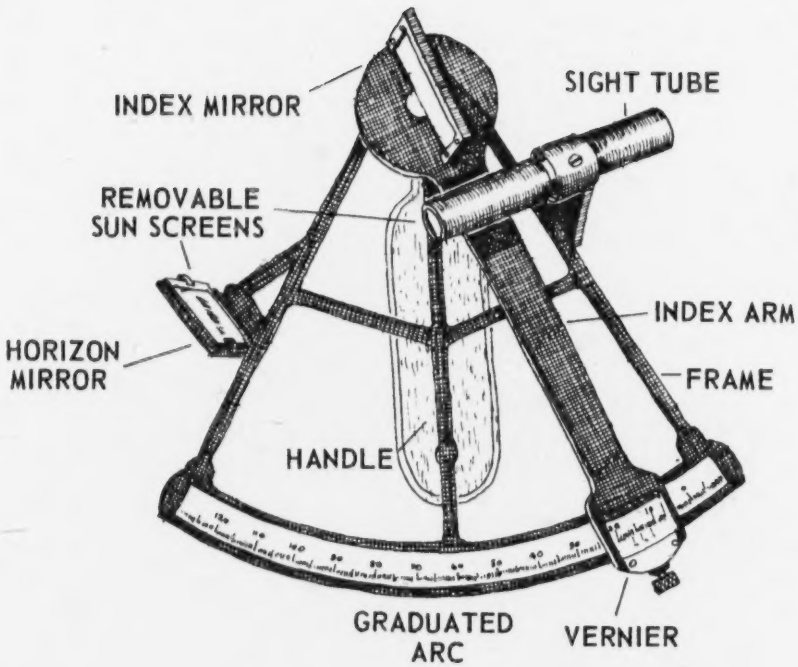
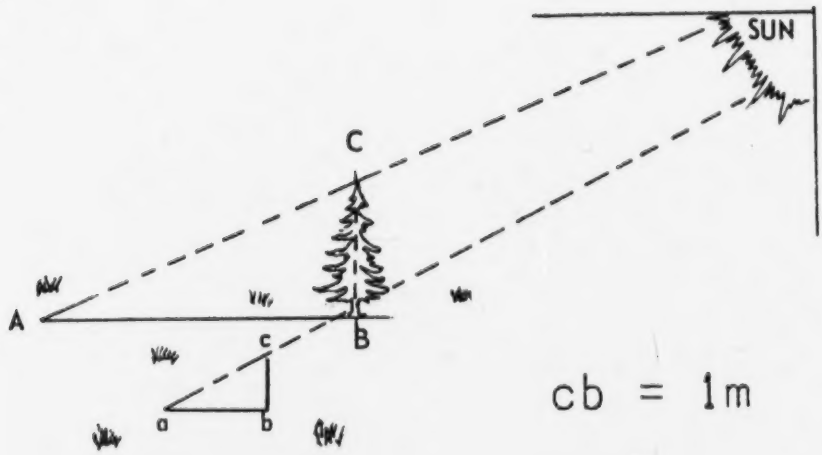


Plate 15



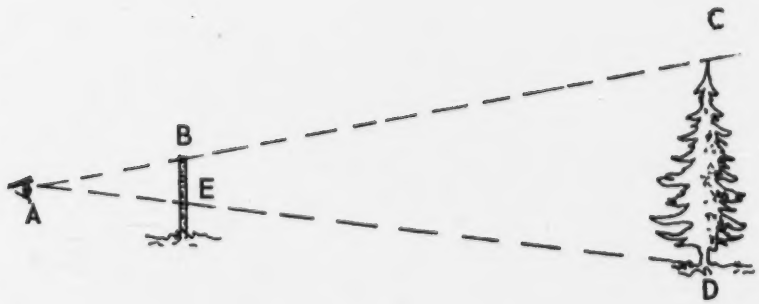
cb = 1m

$\angle a = \angle A$

$\angle B = \angle b = 90^\circ$

$\therefore \triangle ABC \sim \triangle abc$

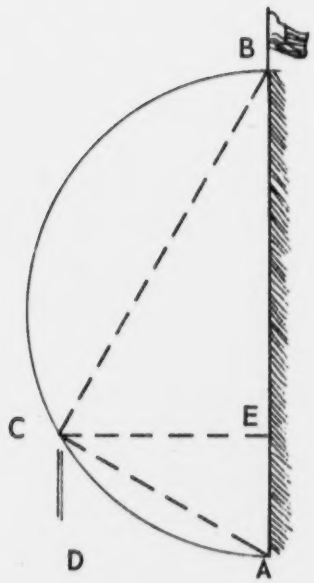
$$\frac{CB}{AB} = \frac{cb}{ab}$$



$\triangle ABE \sim \triangle ADC$

$$\frac{AE}{BE} = \frac{AD}{CD}$$

HEIGHT WITH THE ANGLE MIRROR



$\overline{CE}$ IS THE MEAN PROPORTIONAL BETWEEN

$\overline{AE}$ & $\overline{EB}$

$$\frac{\overline{AE}}{\overline{CE}} = \frac{\overline{CE}}{\overline{BE}}$$

$\overline{AD} = \overline{CE}$

$\overline{AE} = \overline{CD}$

MEASURE $\overline{AD}$ & $\overline{CD}$
COMPUTE $\overline{BE}$

$$\overline{AB} = \overline{AE} + \overline{EB}$$

INACCESSIBLE DISTANCES
ANGLE MIRROR

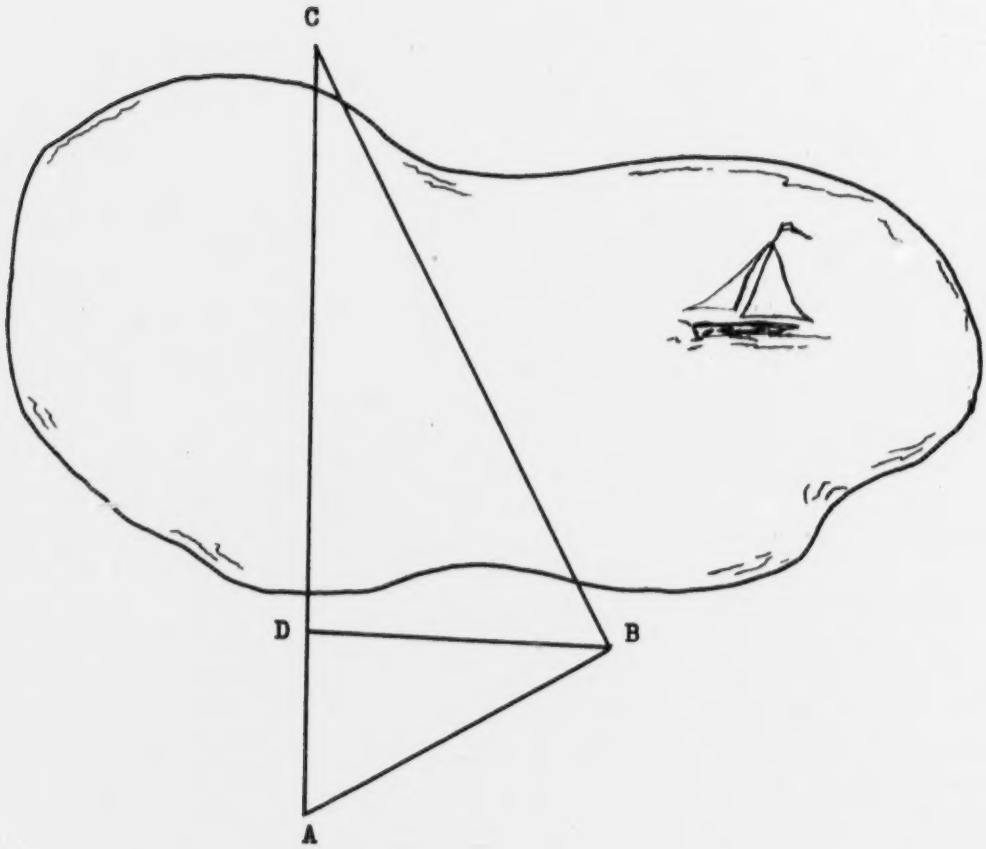


Plate 18

INACCESSIBLE DISTANCES
TRANSIT

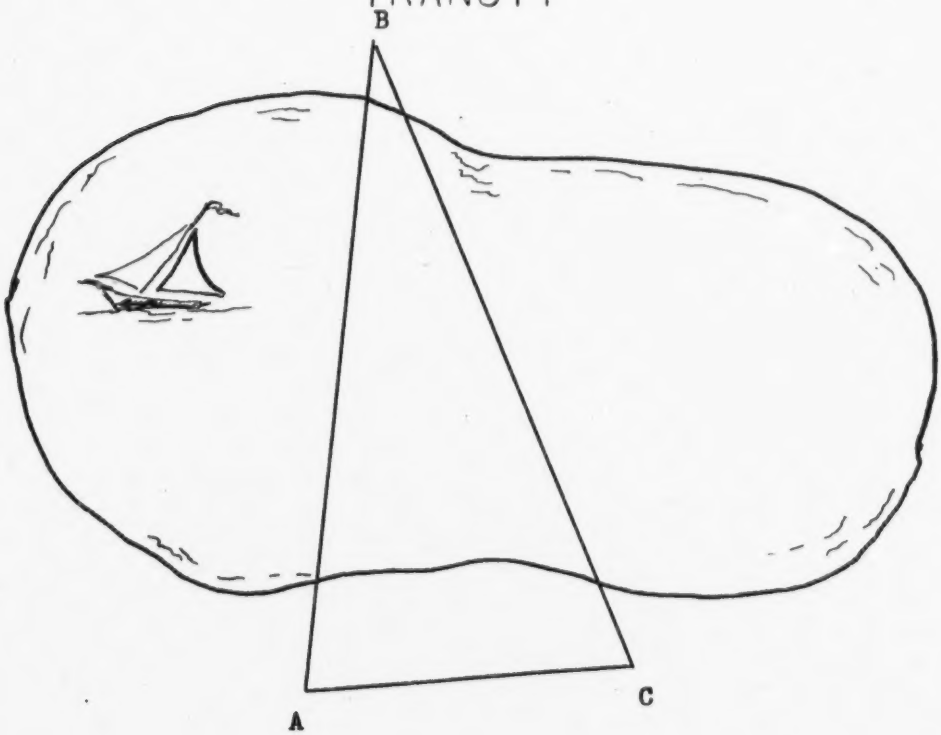
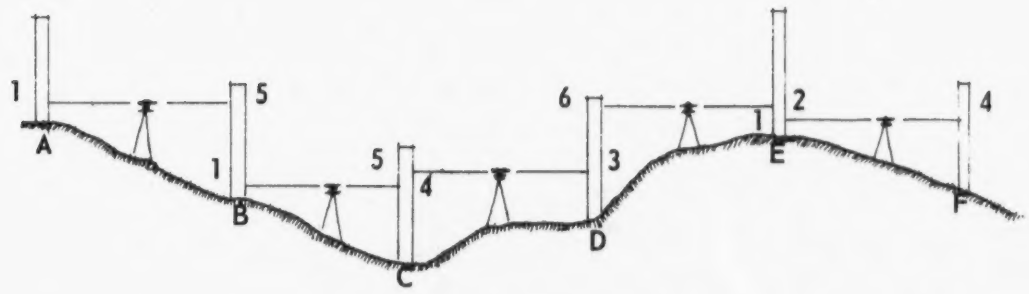


Plate 19

DIFFERENTIAL LEVELING

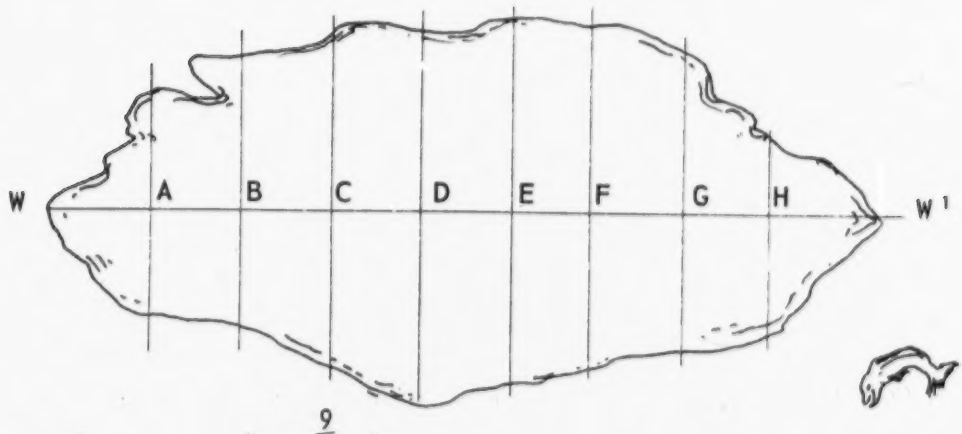


Sta.	+ BS	HI	− FS	ELEV	NOTES
A	1	61		60	BM 60'
B	1	57	5	56	
C	5	58	4	53	
D	6	61	3	55	
E	[illegible]	60	2	59	
F			4	56	

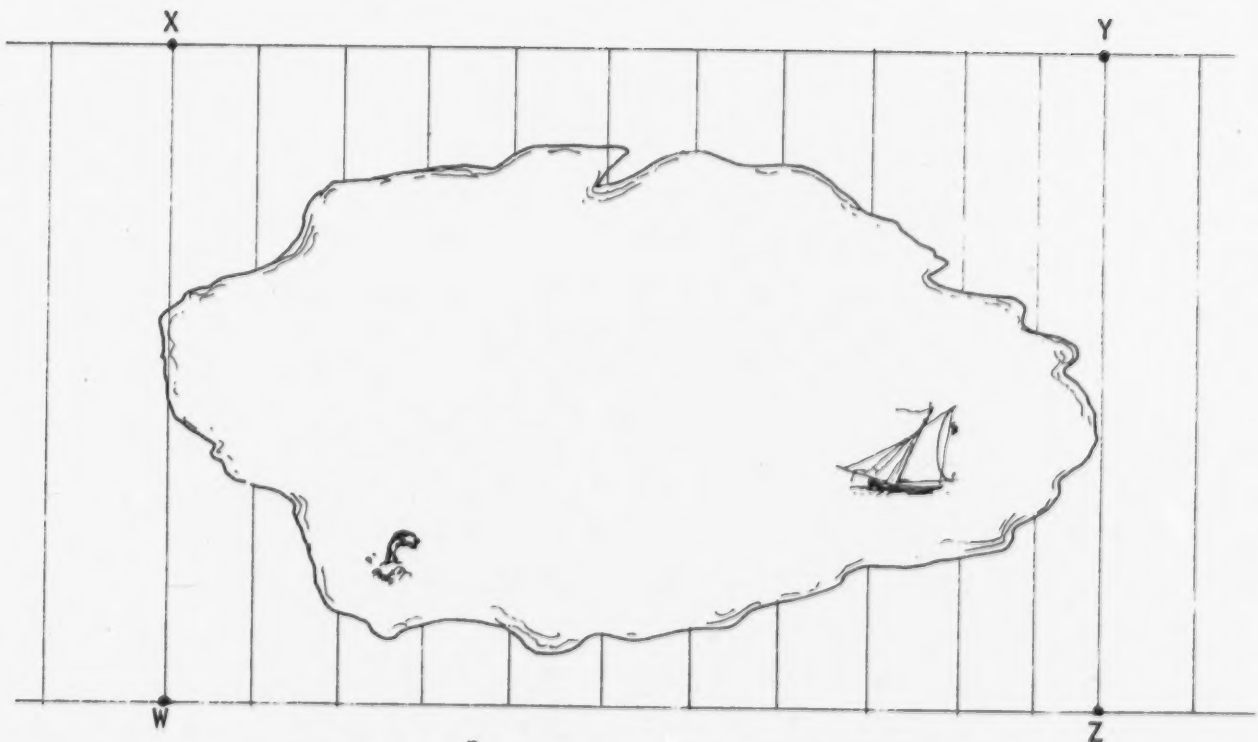
HI = HI of previous location + (BS − FS) on rear station

ELEV = HI − BS
Station ℵ Station ℵ

MAPS AND AREAS



$$A_t \approx \sum_{i=1}^{9} A_i$$

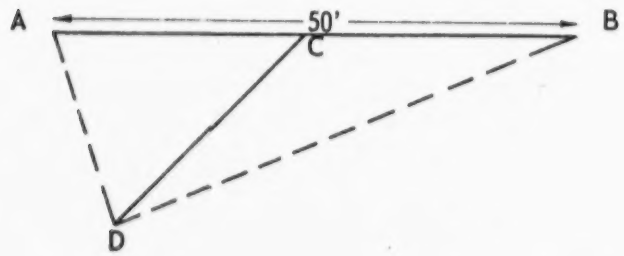


$$A_t \approx A_r - \sum_{i=1}^{n} A_i$$

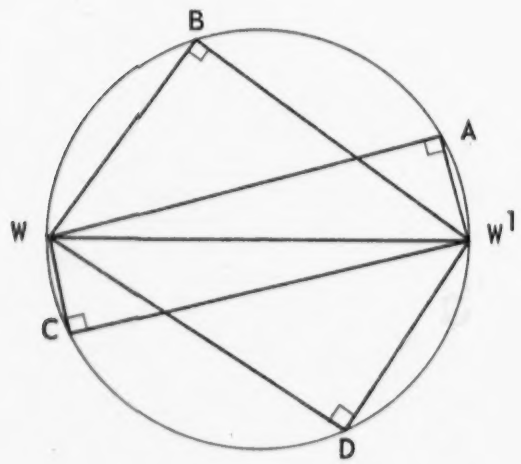
Plate 21

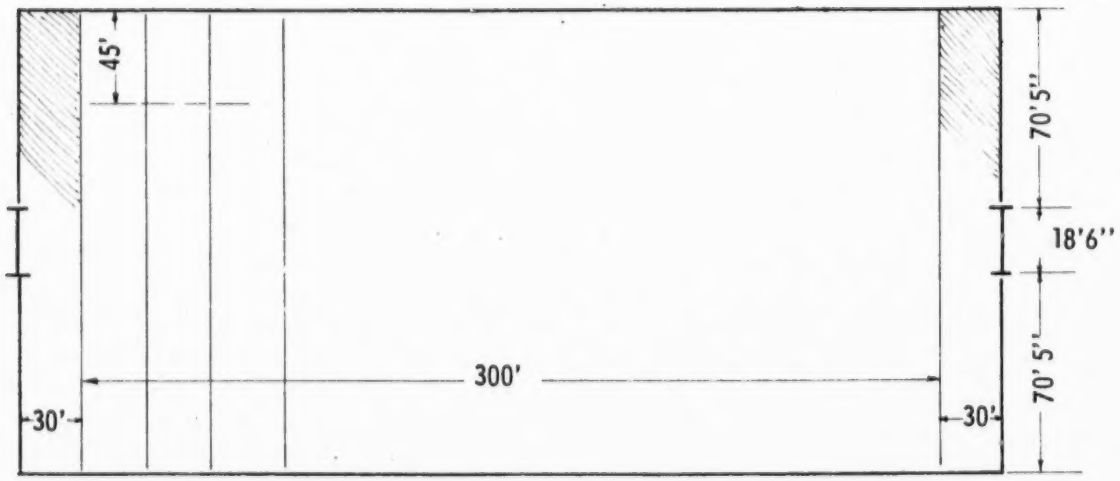
TO CHECK ANGLE MIRROR

A. MARK OFF 50' LINE AB
B. MARK OFF 25' FROM MIDPOINT CD
C. ADJUST MIRROR AT D UNTIL BOTH A&B COINCIDE

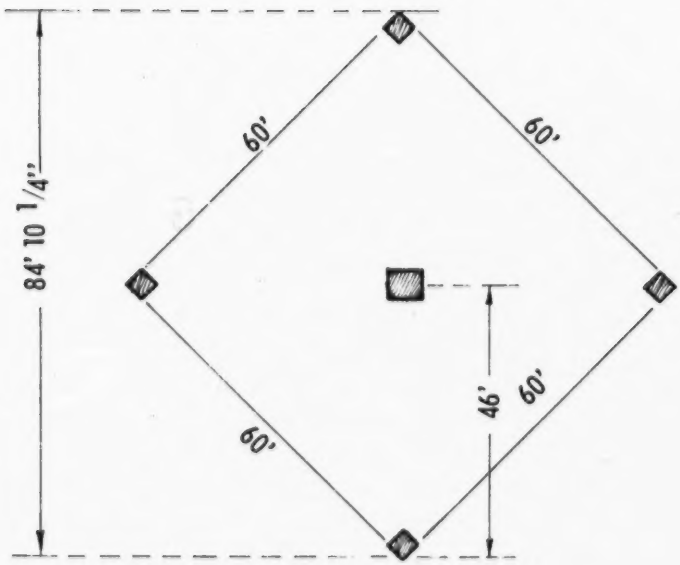


TO LAY OUT A CIRCLE





FOOTBALL



SOFTBALL

Plate 23